Your Signature ____________________

This is a closed book exam. There are six problems. Maximum possible score is 100 points. **Show all your work.** *Partial credit will be given for partial solutions. Correct answers with insufficient or incorrect work will not get any credit.*

Score

1.	(15)	
2.	(15)	
3.	(20)	
4.	(20)	
5.	(20)	
6.	(10)	
Total.	(100)	

Extra sheets attached(if any): ____________________

This is a closed book exam.

If you are using a Theorem then please state it clearly.

Do not assume any results from the homework assignments.

1.(a) Let $f : [a, b] \to \mathbb{R}$. What is meant by saying that "f has Bounded Variation". Give examples of two functions, one which has Bounded Variation and the other which does not.

(b) Let Ω be a non-empty set and $\mathcal{F}$ be a collection of subsets. Suppose $\Omega \in \mathcal{F}$ and that $A, B \in \mathcal{F}$ implies that $A \cap B^c \in \mathcal{F}$ show that $\mathcal{F}$ is an algebra.

2. Let X be a random variable on the probability space $(\Omega, \mathcal{F}, P)$

   (a) Show that X is integrable if and only if $\lim_{n\to\infty} \int_{|X|>n} \mid X \mid dP = 0$.

   (b) Suppose $X \geq 0$, then show that $E(X) = \int_0^\infty P(X \geq x)dx$.

3. Let h_n, f_n, g_n be a sequence of integrable functions on $(\Omega, \mathcal{B}\mu)$ with $h_n \leq f_n \leq g_n$ for all n. Suppose also $h_n \to h$, $f_n \to f$ and $g_n \to g$ on Ω. Show that if the h_n's and the g_n's are integrable and if $\int h_n d\mu \to \int h d\mu$ and $\int g_n d\mu \to \int g d\mu$, then $\int f_n d\mu \to \int f d\mu$

4. Consider two measure spaces $(X, \mathcal{M}, \mu)$ and $(Y, \mathcal{N}, \nu)$, where $X = Y = [0,1]$, $\mathcal{M} = \mathcal{N} = \mathcal{B}_{[0,1]}$, and μ is Lebesgue measure and ν is counting measure. Let $D = \{(x,x) : x \in [0,1]\}$ be in the diagonal in $X \times Y$. Decide which (or if all) of the hypothesis of Fubini's Theorem are satisfied by the function $f = 1_D$.

5. Let $\mathcal{B}_{\mathbb{R}}$ be the Borel σ algebra on $\mathbb{R}$ and $\mathcal{B}$ be any σ-algebra on $\mathbb{R}$ such that every continuous function $f : \mathbb{R} \to \mathbb{R}$ is $(\mathcal{B}, \mathcal{B}_{\mathbb{R}})$ measurable. Then show that $\mathcal{B} = \mathcal{B}_{\mathbb{R}}$.

6. Let $(\Omega, \mathcal{B}, \mu)$ be a finite measure space. Let $f \in L^\infty(\mu)$. Let $\alpha_n = \int_\Omega \mid f \mid^n d\mu$. Show that

$$\lim_{n \to \infty} \frac{\alpha_{n+1}}{\alpha_n} = \| f \|_\infty$$